Probability and Computing 14 Oct, 2014

Problem Set 2

- Due Date: **28 Oct (Tue), 2014**
- Turn in your problem sets electronically (LaTeX, pdf or text file) by email. If you submit handwritten solutions, start each problem on a fresh page.
- Collaboration is encouraged, but all writeups must be done individually and must include names of all collaborators.
- Refering sources other than the text book and class notes is strongly discouraged. But if you do use an external source (eg., other text books, lecture notes, or any material available online), ACKNOWLEDGE all your sources (including collaborators) in your writeup. This will not affect your grades. However, not acknowledging will be treated as a serious case of academic dishonesty.
- The points for each problem are indicated on the side.
- Be clear in your writing.
- 15 problems - 10 marks each

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1. Exercises 3.9, 3.16, 3.19, 3.22, 3.25, 4.9, 4.15, 5.5, 5.7, 5.13, 5.15, 5.16, 5.22 in Mitzenmacher-Upfal

2. Let $X_1, \ldots, X_n$ be Bernouli random variables, not necessarily independent, but satisfying the following weaker condition:

$$\Pr\left[\bigwedge_{i\in S} X_i = 1\right] \leq \prod_{i\in S} \Pr\left[X_i = 1\right], \qquad \forall \text{ subsets } S \subseteq [n].$$

   Prove that under this weaker condition the Chernoff bound holds for $X = \sum X_i$.

3. (sums of exponential variables). Let $Z = Z_1 + \cdots + Z_n$ where $Z_i, i \in [n]$ are independent and identically distributed with the exponential distribution with parameter $\lambda \in (0, 1)$. Recall that the corresponding pdf is $f(x) = \lambda e^{-\lambda x}$ and the cdf is $F(x) = 1 - e^{-\lambda x}$. GIve a sharp concentration result for the upper tail of Z.